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CALCULATION OF THE IMPULSE RESPONSE
IN A TRANSMISSION WHICH IS
TERMINATED BY ITS
CHARACTERISTIC IMPEDANCE
Nils Haaheim
December 1969

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CALCULATION OF THE IMPULSE RESPONSE IN A TRANSMISSION LINE WHICH IS TERMINATED BY ITS CHARACTERISTIC IMPEDANCE

by

Nils Haaheim

Institute for Transmission Techniques

ELAB Report TR-90

August 1967

Technical report

for

Electronics laboratory at NTH associated with SINTEF
Norwegian Technical University
Trondheim

[NTH: Norwegian Technical University]
[SINTEF: Society for Industrial & Technical Research]

INTRODUCTION

- The

This report is the result of an investigation relating to the transmission of data over a cable. The basis of the task consisted in finding out how an impulse that was imposed by means of a current generator on one end of a cable was put out at the other end of the cable. The cable is assumed to be terminated by its characteristic impedance at both ends. It is considered that the primary parameters R and G are frequency dependent. The variations in L and C are so small that the latter have been regarded as constants, and this leads to the non-inclusion of the small roundings-off that in practise occur at the beginning of the impulse. A program has been prepared for calculation of the responses on the GIER calculating machine.

The work has been carried out with financial assistance from the Electronics Laboratory at NTH.

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1. General

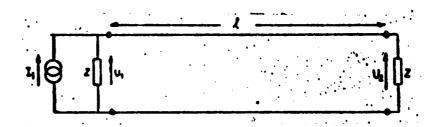
For a homogeneous transmission line one has:
a. the motion-constant

b. characteristic impedans

where R. G. L and C are the line's primary parameters. To a line which is terminated by its characteristic impedance, what is applicable is

$$\frac{d^2}{d^3} = \pi(2r) = n_{-\Lambda \xi}$$

where £ is the line's length.



The line has one unit's impulse imposed

and one obtains the Fourier transformation of the output voltage

$$U_{\mathbf{g}}(\mathbf{j}\omega) = U_{\mathbf{g}}(\mathbf{j}\omega)\mathbf{H}(\mathbf{j}\omega) = \frac{1}{2} e^{-\gamma(\mathbf{j}\omega)\cdot\mathbf{f}}$$

or in the time-plane

$$a^{5}(e) = \frac{5a}{J} \int_{0}^{2\pi} \int_{0}^{\pi} \int_{$$

The Laplace transformation of the output voltage is

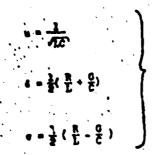
$$v_2(s) = \frac{s}{2} e^{-r(s) \cdot t}$$

2. Frequency-independent parameters

Equation 7 can be inverse-transformed on the assumption that the line parameters are frequency-independent (See for example Weber [WE 1] pages 378-384, and gives

$$u_2(e) = e^{-t \frac{1}{u}} \cdot u(e^{-\frac{t}{u}}) + e^{\frac{t}{u}} = \frac{e^{-t^2}}{\sqrt{e^2 - (t/u)^{2^{-1}}}} I_1[e^{\sqrt{e^2 - (\frac{t}{u})^2}} de]$$

where one has introduced



and $u(t_{\rm U}^{k})$ is unit's impulse for $t=\frac{k}{\rm U}$. Equation 8 is calculated on the digital calculating machine (GIER) and the result of the calculation for a BF-telephone cable of 1 km length and with parameters (See [GR 1] page 29) R = 31.6 ohm/km, L = 0.9 mH/km, G = 2 μ S/km, and C = 0.028 μ F/km is shown in Fig. 2.

For calculation of the modified Bessel-function I, (x) there has been used the serial development of $x^{-1}(x)$ which is applicable to -3.75 < x < 3.75, taken from the "Handbook of Mathematical Functions," Dover 1965, page 378, formula 9.8.3.

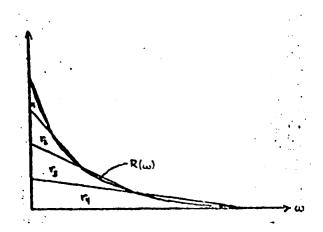
3. Frequency-dependent R and G

For causal time-functions (f(t)=0 for t<0) the time-response can be calculated from the real or the imaginary component of the Fourier-transformation ([PA 1] page 13)

$$F(j\omega) = R(\omega) + jX(\omega)$$

$$f(t) = \frac{2}{\pi} \int_{0}^{\infty} R(\omega) \cos \omega t d\omega = -\frac{2}{\pi} \int_{0}^{\infty} X(\omega) \sin \omega t d\omega$$

Depending on the form of $R(\omega)$, one can choose various methods for numerical calculation of f(t). If $R(\omega)$ is concave, it can be approximated by means of triangles as shown in Fig. 3. [PA 1, page 58]



One then obtains

and the resulting time-response then is

where $g_i(t)$ is the time-function corresponding to $r_i(\omega)$.

We will now find the time-function which corresponds with a spectrum where the real component is triangular as shown in Fig. 4a.

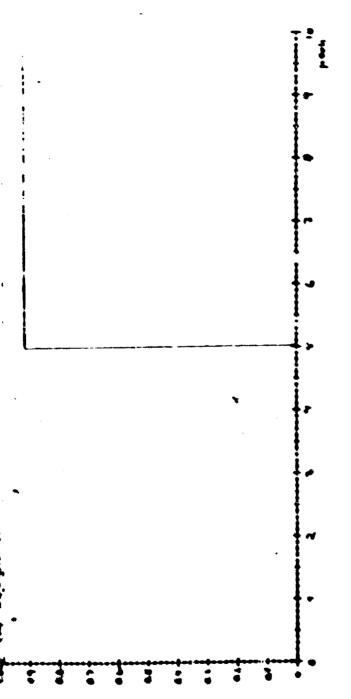
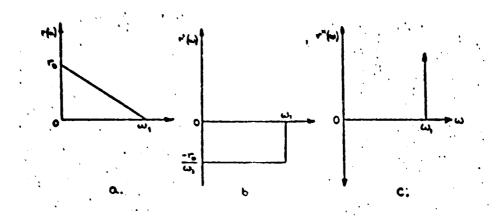


Fig. 2 The impulse response of a cable terminated by its characteristic impedance, when the primary parameters R, L, G and C are frequency-independent.

5.



The derivate, $r^1\left(\omega\right)$, is shown in Fig. 4b and the double-derivate in Fig. 4c.

For a Fourier-pair the applicable equations are

$$f(\varepsilon) \longrightarrow F(\omega) = R(\omega) + j\chi(\omega)$$

$$\varepsilon f(\varepsilon) \longrightarrow j \frac{dP}{d\omega} = jR'(\omega) - \chi'(\omega)$$

$$\varepsilon^2 f(\varepsilon) \longrightarrow \frac{-d^2P}{d\omega^2} = -R''(\omega) - j\chi''(\omega)$$

from which we obtain (See Equation 11)

$$e^{2}f(z) = -\frac{2}{7} \int_{0}^{\infty} R^{2}(\omega) \sin(\omega t) d\omega + 20$$

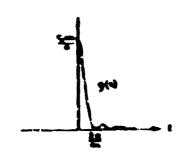
$$e^{2}f(z) = -\frac{2}{7} \int_{0}^{\infty} C^{2}(\omega) \cos(z t) d\omega + 20$$

We now have (See Fig. f)

which gives (Equation 16)

One in that case obtains

This function is illustrated by Fig. 5



7.

In Fig. 6 is shown the course of the real component of the transmission function for a 1.2 mm BF cable of 1 km length and with parameters

L = 0.9 mH/km

 $C = 0.028 \mu F/km$

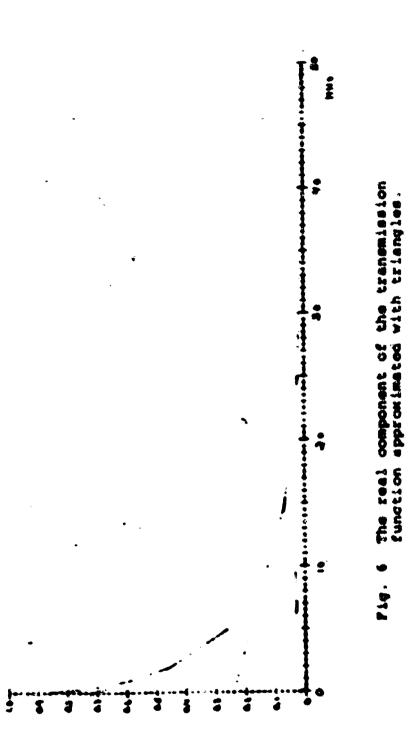
and R and G frequency-dependent

f(kHz)	$R(\Omega/km)$	G(µS/km)				
2	21 6	2				
3 12	31.6 32.2	8.5				
60	43.5	42				
108	56	76				
156	66	111				
204	7 5	144				
252	83	178				
500	117	354				
1000	166	706				
2000 5000	235 370	1412 3530				
10000	525	7000				
15000	640	10500				
25000	830	17600				
50000	1170	35000				

The parameters are taken from Grønlie [GR 1] page 29 and several values are calculated from G being proportional to f and R being proportional to \sqrt{f} .

As appears from Fig. 6, the real component is concave and can be approximated with a triangle, as mentioned. The term which represents the constant phase-contribution is deducted, but it is taken into consideration later when calculating the impulse response. The sum of the inverse-transformations of individual contributions from the triangles gives the cable's impulse response.

On numerical integration of this sum, one finds thus the impulse response of the cable.



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4. Program descriptions

Three different programs have been written in GIER algol III.

4.1. Calculation of the impulse response in a cable on the assumption of constant parameters.

The method is described in Para. 2.

Data strip:

R(ohm/km)

L(H/km)

G(S/km)

C(F/km)

cable length in km

the length of the time-axis in secs.

The result is obtained on the plotter on GIER I. The pen must be placed 1-2 cm from the left edge of the sheet before the calculation begins. Program write-out, or transcript, and example are appended. Procedure linscale is utilized in the program and does appear on the strip, but not on the transcript. An example is shown in Fig. 2.

4.2. Calculation of the real component of the transmission function.

The program is utilized to calculate the real component of the transmission function

 $R(x) = Au(a^{-\gamma(x)} \cdot f)$

and can be utilized to obtain an impression of how many frequency values one will use for the next program and how the latter will be chosen.

Data strip:

L(H/km)

C(F/km)

cable length in km

number of frequency values

$$f_1(Hz)$$
 , $R_1(ohm/km)$, $G_1(S/km)$

 f_2 , R_2 , G_2

and so on.

The result is plotted on the plotter connected to GIER I.

An example is shown in Fig. 6.

The program transcript is appended, but procedure linscale is not included in the transcript.

4.3. <u>Calculation of the impulse response when R and G are frequency-dependent</u>.

The program calculates the impulse response in accordance with the method which is described in Para. 3.

Data strip: (See also Para 4.2.)

L(H/km)

C(F/km)

cable length in km

the step-length for integration in sec.

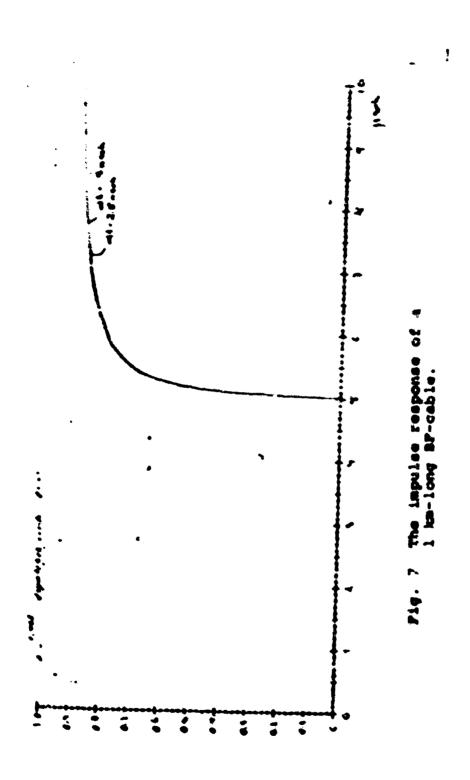
the length of the time-axis in sec.

number of frequency values

$$f_1(Hz)$$
 , $R_1(ohm/km)$, $G_1(S/km)$
 f_2 , R_2 , G_2
and so on.

The result is plotted on the plotter connected to GIER I. In order to examine the accuracy of the integration, one needs to work with two different step-lengths as shown in the following example where one has used step-length 5 nsec. and 2.5 nsec. Transcription of the data-strips appears on the next page and the result of the calculation in Fig. 7. As appears from the figures, the derivations between the two curves are very small, and the result will be satisfactory for most practical purposes. Here one will also have the benefit of the program which calculates the course on the assumption of frequency-independent parameters (Para. 4.1.) which gives the asymptotic course. On the typewriter four numbers are transcribed, namely:

- 1. The time-delay
- 2. The stationary value of the impulse
- 3 and 4 are real value for the transmission function in the case of the first and last frequency.



13.

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Data strip:

tmax = +1 sec

Number of frequency values = 15

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Data strip:

than en sec

Number of frequency values = 15

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The impulse response of a cable, on the assumption that the primary parameters R, L, G and C are frequency-independent.

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The real component of the transmission function.

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Data strip:

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Number of frequency values = 15

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The impulse response in a transmission line which is terminated by its characteristic impedance. R and G are frequency-dependent.

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5. Conclusion

The calculations show that direct current signalling with high speeds is possible over an appropriate cable. However, the long transient-times will shift the "O" line independently of the duration and distance of the signal pulse. It may therefore be necessary to carry out a correction; the response for the lower-frequencies especially has to be reduced. A correction network for a cable is dealt with in a thesis for the Institute for Transmission Techniques, NTH. (SA1)

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